CSCI 2570 Introduction to Nanocomputing

Encoded NW Decoders

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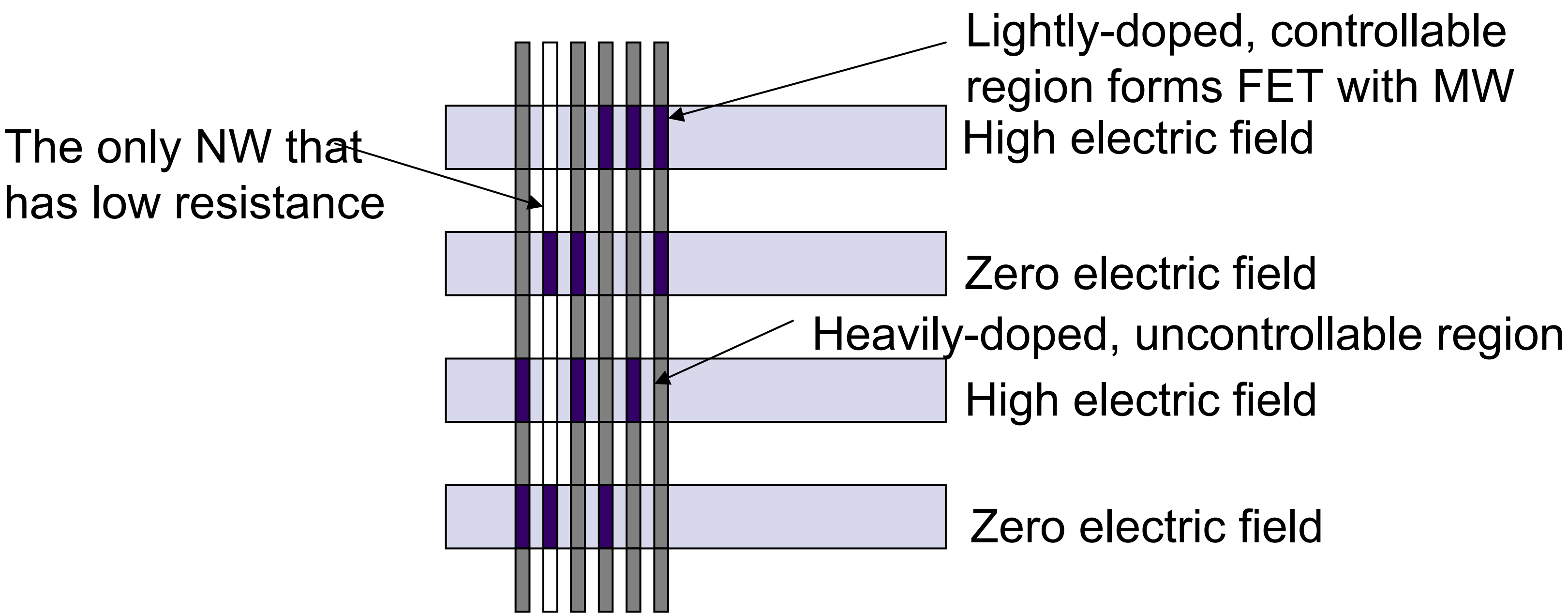
Lecture Outline

- Encoded NW Decoders
  - Axial and radial encoding
- Addressing Strategies
  - All different, Most different, All present, Repeated codeword, and "Take What You Get"
- Wildcarding – addressing multiple rows/cols
- Codeword Discovery

Axially Encoded NWs

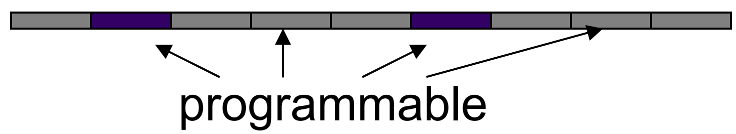
- NWs controlled by MWs



- What NW codes should be used?

Axial Nanowire Codes

- ***(h,M)*-hot encoding**
  - *M* programmable regions *h* of which are lightly doped. $C = \binom{M}{h}$ codewords.



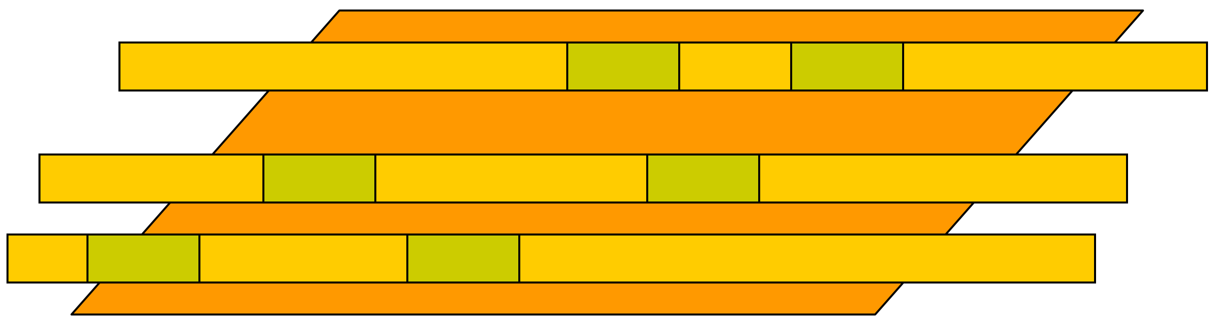
- ***M*-bit binary reflected codes**
  - $(x_1, x_2, \ldots, x_{M/2}, \overline{x}_1, \overline{x}_2, \ldots, \overline{x}_{M/2})$ denotes programming in programmable regions, $y = 1\ (0)$ is lightly (heavily) doped. $C = 2^{M/2}$ codewords.

Comparison of Axial Codes

- *(h,M)*-hot codes can have more codewords than *M*-bit binary reflected codes if *h* is close to *M/2* but not if *h* is small, say, *h*=2
- *(h,M)*-hot codes have $C = \binom{M}{h} \approx \frac{2^M}{\sqrt{\pi M/2}}$ when *h* = *M/2*, whereas *M*-bit binary reflected codes have $C = 2^{M/2}$ codewords.
- However, binary reflected codes have natural mapping from binary tuples.

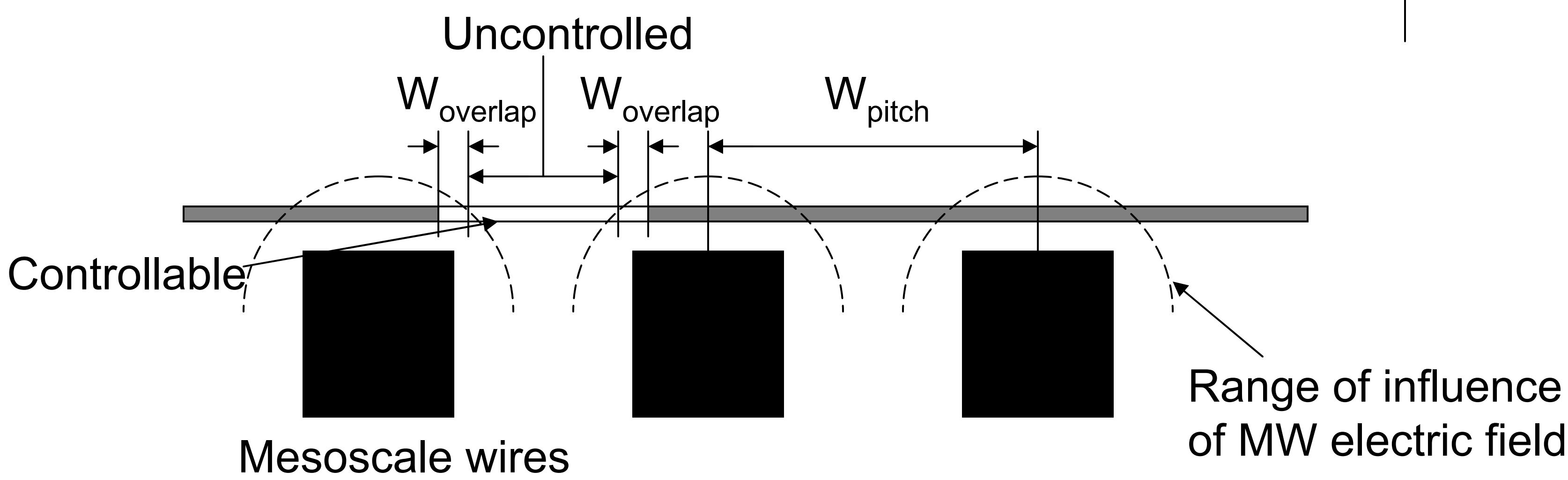
Fluidic Assembly of Differentiated NWs

- Random sample of coded NWs is floated on a liquid, deposited on chip, and dried.



- NWs self-assemble into parallel locations.
- Process repeated at right angles – crossbar.

The Effect of Misalignment on NW Controllability



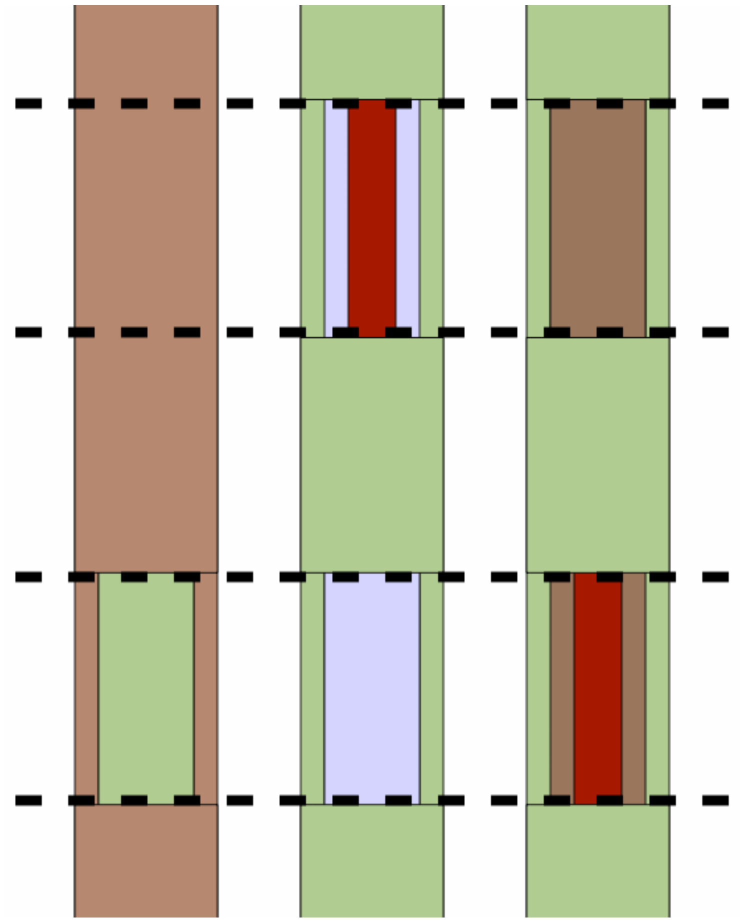
- $W_{overlap}$ is length over which field is ambiguous.
- Probability that a NW is controlled by MW:

$$P_{control} = (1 - 2W_{overlap}/W_{pitch})$$

- Need to detect uncontrollable NWs.

Core-Shell NWs Radial Encoding

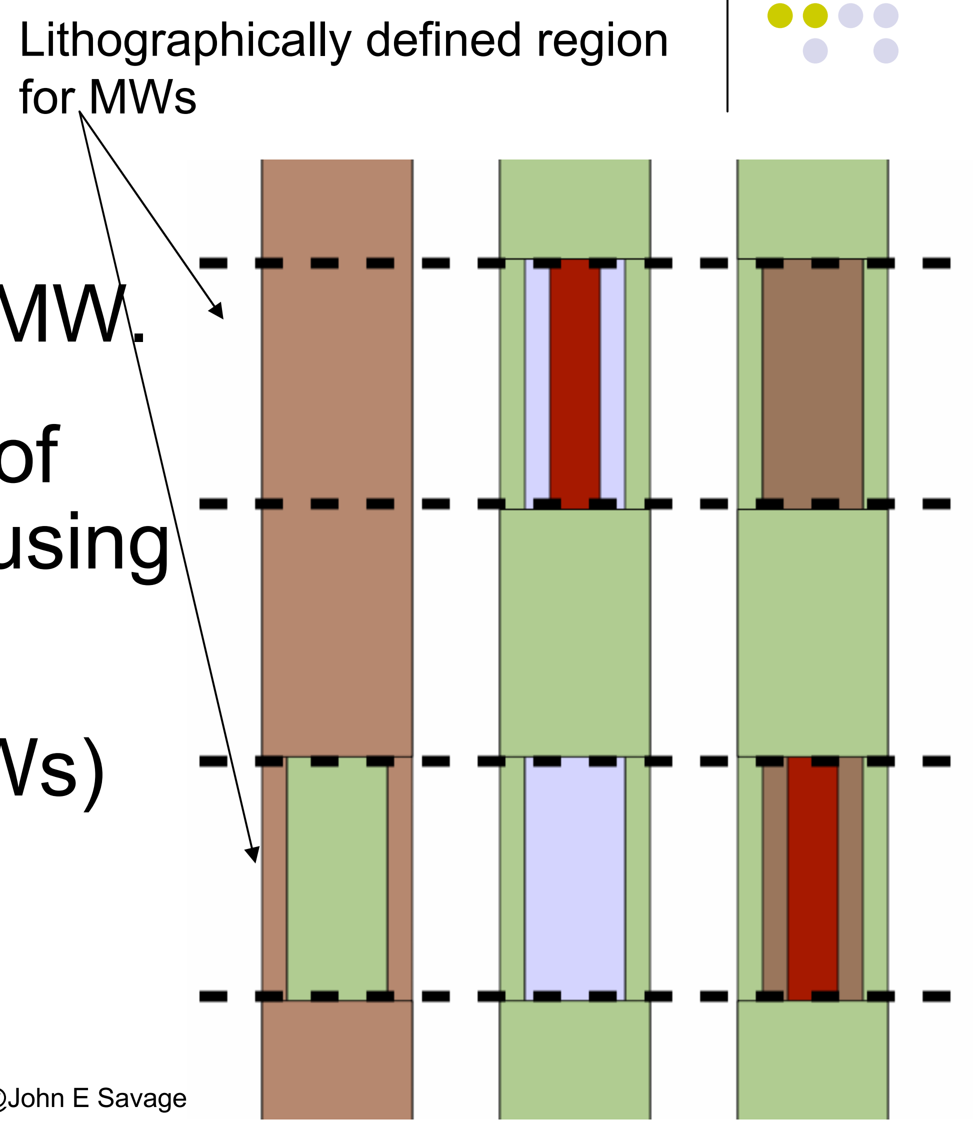
- Shells put on lightly doped NWs
- Shells made of differentially etchable material.
  - One material can be removed by etching without affecting the other materials



Selective Etching of Core-Shell NWs

- Consider NWs with the shell sequence $\mu_1, \ldots, \mu_s$. Here μ_1 is the outer shell.
- Let $E(\mu_i)$ be the etching process that removes only material μ_i.
- The etching sequence $E(\mu_1), \ldots, E(\mu_s)$ exposes only the cores of NWs with the shell sequence $\mu_1, \ldots, \mu_s$.

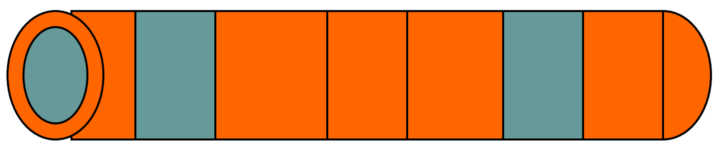
Linear Decoder for Core-Shell NWs



- Apply *s* step etching sequence under each MW.
- $C = m(m - 1)^{(s-1)}$ types of NW can be controlled using $M = C$ address wires
- *12* codewords (and MWs) suffice to control 1,000 NWs for $w = 10$!

Outline of Logarithmic Decoder

- When $s = 1$, each NW can be etched with an arbitrary codeword. Under each MW do an etch if the shell type is in some set.



- Any encoding can be extended to s shells if materials in consecutive layers are different.
- This limits C to at most $(m/2)^s$.

Single-Shell Logarithmic Decoder for Radially Encoded NWs

- Assign binary *L*-tuple to each one of **α shell types** where $L = \lceil \log_2 \alpha \rceil$ Let $M_{0,i}$ and $M_{1,i}$ be materials with 0 and 1 in *i*th bit of their representations $1 \leq i \leq L$
- Use *2L* MWs $W_{0,i}$ and $W_{1,i}$ Under $W_{b,i}$ remove materials in $M_{b,i}$
- To leave NW with shell tuple E on, apply fields to all MWs that do not control the NW.
- Decoder uses $2\lceil \log_2 \alpha \rceil$ MWs. How many etchings?

Multi-Shell Radially Encoded NWs

- Partition α shell types into two sets β_1, β_2, of size α_1 and α_2, $\alpha = \alpha_1 + \alpha_2$. In alternate shells use materials β_1 and β_2.
- With *n* shells, there are $C = (\alpha_1\alpha_2)^{(n/2)}$ NW encodings for *n* even.

Decoders for Multi-Shell Radially Encoded NWs

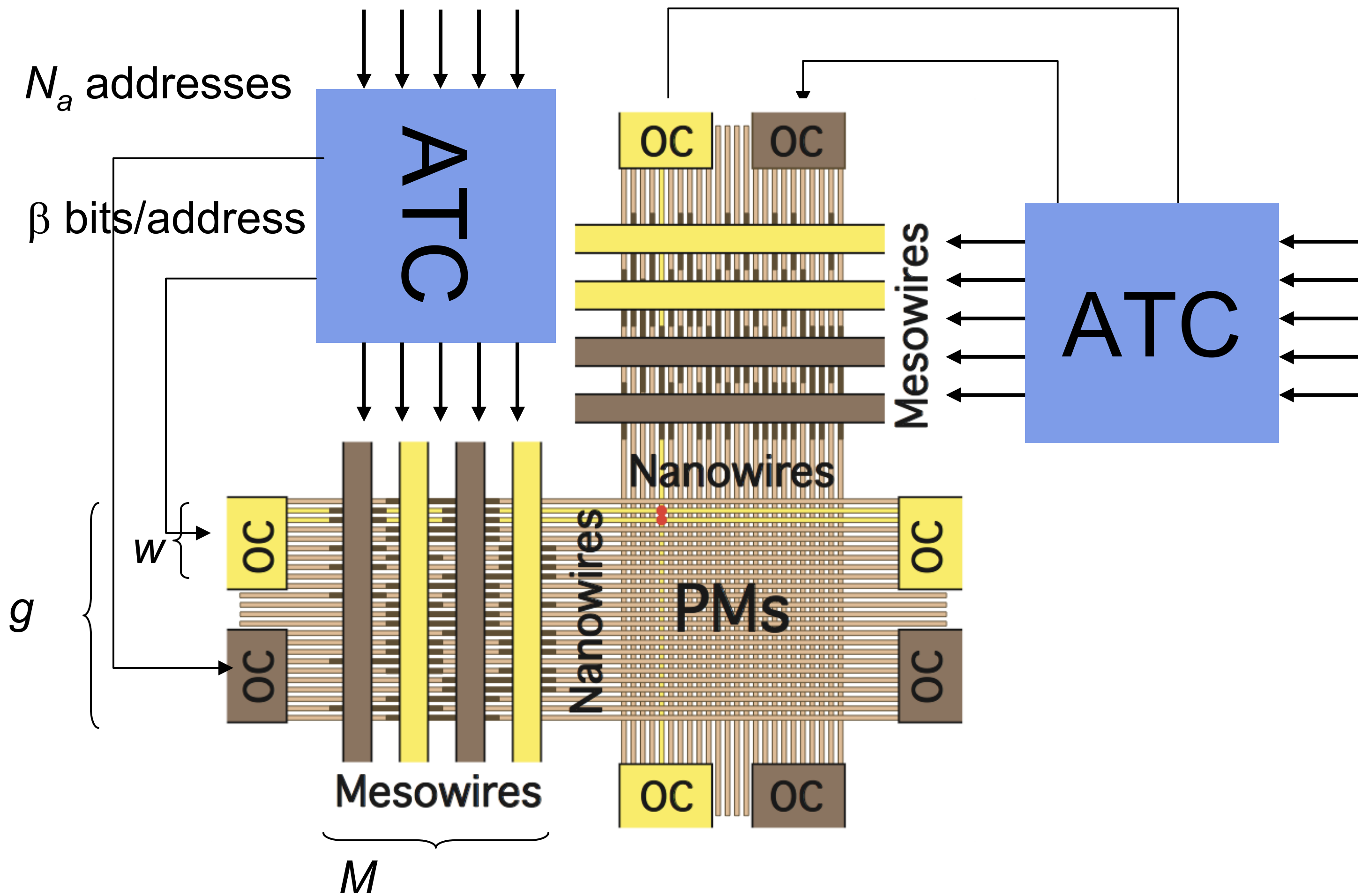
- Let *LayerEtch(M,W,s)* remove all materials in the first *s-1* shells, those in set *M* in shell *s,* and all materials in higher shells, if exposed.

- When combined with the logarithmic decoder, it uses $M = n\ (\log_2 (\alpha_1) + \log_2 (\alpha_2)\ )$ MWs, *n* even.

- How many etchings does it use?

Types of Simple Decoder

The Crossbar Memory



Crossbar Parameters

- $N = gw$ = no. NWs per dimension
  - g = no. of contact groups
  - w = no. of NWs per ohmic region
- C = no. of codewords – desirable keep small
- M = no. MWs per dimension – ditto
- β = no. bits to address each NW
  - $\beta = M + \lceil \log_2 g \rceil$ (not for AWA) for *(h,M)-hot codes*
  - $\beta = M/2 + \log_2 g$ for BRCs
- N_a = no. of addressable NW types/dimension

Area Estimates of Crossbar Memories

- σ = area per ATC bit
- N_a = number of addressable NWs per dimension
- $(N_a)^2$ addressable crosspoints
- $2\lambda_{meso}$ = pitch of MW
- $2\lambda_{nano}$ = pitch of NW

- Area of 2 standard decoders = $2\lambda_{meso}\, g \log_2 g$
- Area of 2 ATCs = $2\sigma\beta N_a$
- Area of array = $4(M\lambda_{meso} + N\lambda_{nano})^2$

- $A_T \approx 2\sigma\beta N_a + 2\lambda^2_{meso}\, g \log_2 g + 4(\lambda_{meso}M + \lambda_{nano}N)^2$

Goals of Addressing Strategies

- Minimize chip area given N_a individually addressable NWs with probability ≥ 1-ε.

- Note:
  - Probability that N_a is large increases with C
  - Size of translation memory grows with C
    - For radial encodings, effective NW pitch and area will grow with number of shells
    - For axial encodings, loss of NWs due to misalignment
  - Addressing strategy also affects chip area and N_a

NW Addressing Requirements Examined

- All wires addressable in each contact group
- Most wires addressable in each group
- Take What You Get
  - Use all individually addressable NWs

Bounds on Probabilities

Lemma 1 Prob. that each of the *w* NWs in a contact group has distinct encoding satisfies

$$P_{distinct} \leq e^{-w(w-1)/2C}$$

Thus, $P_{distinct} \geq 1-\delta$ when $C \geq w(w-1)/(-2\ln(1-\delta))$

Proof New code on 1st trial. New code on *j*th trial with probability (*1-(j-1)/C*). All codes are different with probability $\prod_{j=2}^{w}(1-(j-1)/C)$ Using $(1-x) \leq e^{-x}$ and $\sum_{j=1}^{w-1} j = w(w-1)/2$ the result follows.

All Wires Addressable in Each Contact Group

Theorem Strategy succeeds with prob. 1- ε when

$$C \geq N_a(w-1)/(-2\ln(1-\epsilon)) \approx N_a(w-1)/(2\epsilon)$$

Proof Let δ be prob of failure to have all NWs be distinct in one contact group. Prob. that strategy succeeds is $(1-\delta)^g = 1-\varepsilon$. When ε is small, $\delta \approx \varepsilon/g$. Result follows from Lemma 1.

Performance of All Wires Addressable

- No wasted NWs
- Very large value for C
  - $C \geq N_a(w-1)/(2\varepsilon)$
- $\beta = 2 \log_2 C$ and $N_a = N = gw$

$$A_T \approx 4\sigma N_a \log_2 C + 2\lambda^2_{meso}\, g \log_2 g + 4(2\lambda_{meso} \log_2 C + \lambda_{nano} N_a)^2$$

Coupon Collection for Most Wires Different Strategy

Lemma 2 Let $2d \geq w \geq 8$. No. different NW encodings, C, needed for d of w NWs to be unique with probability $\geq 1-\delta$ satisfies

$$C \geq (d-1)e^{((d-1)-\ln\delta)/(w-d+1)}$$

Proof Failure if $\geq k = (C-d+1)$ codewords are missing or $\leq d-1$ present. Let Q be this prob. Let event $F_{\boldsymbol{c}}$ be codewords $\boldsymbol{c}$ in $\{c_1, \ldots, c_k\}$ doesn't occur in w trials. $Q = \Pr(E)$ where $E = \cup_{\boldsymbol{c}} F_{\boldsymbol{c}}$. Use Inclusion/Exclusion.

Coupon Collection for Most Wires Different Strategy

Proof $\sum_c P(F_c) - \frac{1}{2}\sum_{c_1 \neq c_2} P(F_{c_1} \cap F_{c_2}) \le P(E) \le \sum_c P(F_c)$

$P(F_c) = (1 - \frac{k}{C})^w$. If $F_{c1} \cap F_{c2}$ have s words in common, $P(F_{c_1} \cap F_{c_2}) = (1 - (2k - s)/C)$

There are $\binom{C}{k}\binom{k}{s}\binom{C-k}{k-s}$ ways to choose c_1 and c_2 to meet this condition. Upper & lower bounds are close.

Since there are $\binom{C}{k}$ ways to choose **c**, we have upper & lower bounds close to $Q \le \binom{C}{k}(1 - \frac{k}{C})^w = \binom{C}{d-1}(\frac{d-1}{C})^w$

Approximate it and make ≤ ε.

Coupon Collection for Most Wires Different Strategy

Let $H(x) = -x \ln x - (1-x)\ln(1-x)$ Then, $\binom{C}{a} \le e^{CH(a/C)}$. Using $-\ln(1-x) \le x(1+x)$ for $x \le .65$ we have $\binom{C}{d-1} \le (\frac{d-1}{C})^{d-1} e^{d-1}$ when C > 1.54(*d*-1). The result follows *d* = *(w+1)/2. Q.E.D.*

Most Wires Addressable in Each Contact Group

- ≥ $(w+1)/2$ different NWs in each contact group

Theorem Strategy succeeds with prob 1-*ε* when where $C \geq e^{(w-2\ln\delta)/(w+1)}(w-1)/2$

Proof Follows directly from Lemma 2 and

$$\ln(1-\delta) = \ln(1-\epsilon)/g$$

- When *ε* = .01, 15 ≤ C ≤ 30 for 10 ≤ m ≤ 500, 10 ≤ *w* ≤ 20.

Performance of Most Wires Addressable (MWA)

- About half of NWs wasted. $N_a \geq N/2$

$$C \geq e^{(w-2\ln(\epsilon/g))/(w+1)}(w-1)/2$$

$$A_T \approx 4\sigma N_a \log_2 C + 2\lambda^2_{meso}\, g \log_2 g + (2\lambda_{meso}\log_2 C + 2\lambda_{nano}N_a)^2$$

- **Comparison**: All terms same except for 2x. However, C much smaller for MWA.
  - When $\varepsilon = .01$, $C_{awd} \approx 50N_a w$ but $C_{mwa} \approx 3.14(w\text{-}1)g^{0.182} \leq 15\,w$ for $w \geq 10$ and $m \leq 5{,}000.$

"Take What You Get" Strategy

- Analyze number of different NW codewords using Hoeffding's Inequality. Let $S = n_1 + \ldots + n_t$ where $\{n_i\}$ are ind. r.v.s in $a_i \leq n_i \leq b_i$. For $d > 0$ and $c_i = b_i - a_i$.

$$P\left(E[S] - S \geq d\right) \leq e^{-2d^2/\sum c_i^2}$$

"Take What You Get" Strategy

$$P\left(E[S]-S \geq d\right) \leq e^{-2d^2/\sum c_i^2}$$

Theorem Let N_a be total no. addressable NWs in a decoder with g contact groups, w NWs per group, and $N = gw$ NWs.

$$P(N_a \leq E[N_a]-Nk) \leq e^{-2k^2 Nw/(w-1)^2} = e^{-2k^2 g^*}$$

for $k > 0$ and $g^* = g(w/(w-1))^2$.

Proof Let $t = g$, $d = Nk$, $S = N_a$ and $c_i = (w-1)$.

"Take What You Get" Strategy

Corollary Let N_a be total number of addr. NWs in decoder with g groups, w NWs/group $N = gw$ total NWs, M MWs, $g^* = g(w/(w-1))^2$

$$P(N_a > \kappa N) \geq 1 - \epsilon$$

if $\kappa \leq \alpha(1 - e^{1/\alpha}) - k$ where $k = \sqrt{\frac{-\ln \epsilon}{2g*}}$

Proof Clearly $E[N_a] = C(1 - (1 - 1/C)^w)$

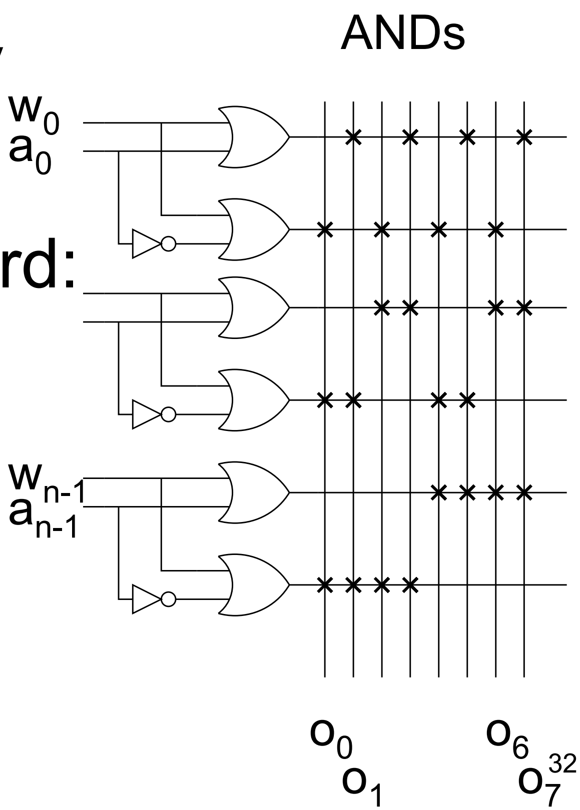
assume $C = \alpha w,\ \alpha > 1$ Using $(1 - 1/C) \approx e^{-1/C}$

and $e^{-2k^2 g*} = \epsilon$ gives value for k and κ

Note: If g=230, N=1,380, ε=.01, M=8, $\kappa N = 1{,}018$

Wildcarding

- Goal: read or write bits in groups
  - Useful in codeword discovery
- Augment memory address by wildcard bits
  - Address: $(a_0, a_1,\ldots,a_{n-1})$; Wildcard: $(w_0, w_1,\ldots,w_{n-1})$
  - If $w_i = 1$, addresses with both values of a_i are used



Codeword Discovery for All Wires Addressable

- Test for presence of codeword by writing 1 and then reading to see if stored.
  - Can activate all NWs in an "orthogonal" group
- Wildcarding writes multiple 1s or 0s.
  - Most useful when most addresses absent, as in Most Different and All Different Strategies
- Reading is equivalent to ORing of data stored.
  - 1 returned if any bit that is read is 1.

Searching Code Space for All Wires Different Strategy

- All b-bit codewords likely to be unique
- Number of words in code space = $50N_a w$!
- Procedure:
  1. Write 1s to all addresses in a contact group
  2. With wildcarding, read addresses with l.s.b. 0
  3. If successful, fix bit and repeat on other bits
  4. If unsuccessful, repeat with l.s.b. 1.
  5. When all bits found, set stored value to 0 and repeat
- $\leq$ b steps/discovered codeword, b = $\log_2 C$ for BRC
  - $N_a \log_2 (2C)$ steps, much smaller than exhaustive search.

Lower Bounds on Discovery Time

- Assume ≥ αw unique codewords/group
- ≥ $\binom{C}{\alpha w}$ ways to choose codewords/group
- ≥ $\binom{C}{\alpha w}^g$ ways to choose codewords/dimension
- Since each read output is binary, $g \log_2 \binom{C}{\alpha w}$ reads are needed to discover all codewords
- But $\binom{C}{\alpha w} \geq (C/(\alpha w))^{\alpha w}$ so $N_a \geq \log_2(C/(\alpha w))$ steps needed.
- Compare with upper bound $N_a \log_2 4C/(w+1)$

Citations

Stochastic Assembly of Sublithographic Nanoscale Interfaces by André DeHon, Patrick Lincoln, John E. Savage , *IEEE Transactions in Nanotechnology*, September 2003.

Evaluation of Design Strategies for Stochastically Assembled Nanoarray Memories, Benjamin Gojman, Eric Rachlin, and John E. Savage, ACM J. on Emerging Technologies in Computing Systems, Vol. 1, No. 2, pp. 73-108, July 2005.

Conclusions

- There are many ways to encode and decode NWs!
- There are many problems to be solved to make nanoarrays practical.

"Take What You Get" Strategy

Use all available NWs

S_{rc}

Strategy					
1	N	C	N_a	N_a^2	$2N_a \log_2(mC)$
	350	10	213	45,369	3,834
	350	15	246	60,516	4,920
	350	20	265	70,225	5,300
	350	30	287	82,369	6,314
	1,000	10	627	393,129	12,540
	1,000	15	722	521,284	15,884
	1,000	20	778	605,284	17,116
	1,000	30	839	703,921	20,136
2	N	C	N_a	N_a^2	$2N_a \log_2(m)$
	350	10	140	19,600	1,680
	350	15	120	14,400	1,440
	350	20	100	10,000	1,200
	350	30	60	3,600	720
	1,000	10	500	250,000	7,000
	1,000	15	510	260,100	7,140
	1,000	20	480	230,400	6,720
	1,000	30	420	176,400	5,880

Result of 100,000 runs

Figure 1: Summary of Monte Carlo experiments in which there are $w = 10$ NWs per ohmic region and $N = mw$ NWs in m ohmic regions. Shown as a function of N and C are the values of N_a, N_a^2, and the storage capacity of an auxiliary translation memory, which is $2N_a \log_2(mC)$ for the first strategy and $2N_a \log_2(m)$ for the second.

"Small Codespace Addressing Strategies for Nanoarrays" by E. Rachlin & J.E. Savage, CS NanoNote #3, May 31, 2005